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PREDICTING INVERSION DEPTHS AND TEMPERATURE INFLUENCES IN THE HELENA VALLEY

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I. INTRODUCTION

Surface inversions in the Helena valley are common throughout the year with strongest and most frequent occurrence during summer and fall. The quantitative effect of inversions has been imperfectly understood because the depth and configuration have been unknown. The purpose of this paper is to present an objective method for predicting the depth of the local inversion and its influence on the maximum or hourly temperature using a remote temperature sounding (Great Falls, 90 miles away) and the local minimum temperature. This is accomplished by assuming the depth of the inversion under clear skies and calm winds is directly related to the minimum temperature and the "temperature" which would occur if no inversion formed.

II. DEVELOPMENT OF THE INVERSION DEPTH FORMULAE

From summer of 1970 to March 1971, the National Weather Service Office, Helena, received aircraft temperature soundings from surface to about 8000 feet MSL during mornings when it was suspected an inversion existed. However, no aircraft sounding was ever taken on Sunday (day of rest!). Soundings were obtained from a local flying service aircraft originally hired by American Smelting and Refinery of east Helena, and after September 1970, the National Health Service. Temperature sensors used were the aircraft's own thermometer prior to September 1970, and a thermistor loaned by Western Region Headuqarters, National Weather Service, after September 1970.

Careful studies of some of the aircraft soundings (see Figure 5) indicate that local inversions often resemble the exponential curve.

(1) $Z = a^{\Delta T} - 1$

where: Z = Equivalent height above the surface in C° units on an adiabatic chart.

ΔT = Temperature difference from the minimum (T_{MI}) to any temperature (T) in C° (T_{MI} will therefore always be the origin for equation (1)).

a = a constant.

Furthermore, the area (energy loss) under the inversion appears to be directly related to the difference between the surface temperature which would occur should no inversion exist, T_L, and the observed surface minimum T_{MI}. (See Figure I.) In fact, 22 fall aircraft soundings considered ideal for the purpose revealed through linear regression techniques that:

(2) $A^I = 3.5 (T_L - T_{MI})$

where: A^I = Area in C° square units on adiabatic chart D-11.

T_L = That temperature which would result if the sounding above the inversion were extended down to the surface.

T_{MI} = Observed minimum surface temperature.

Equation (2) is really the crux of the entire presentation. We know that an inversion forms by radiative and transfer processes; and since the surface temperature change from T_L to T_{MI} is a balance of these processes, it seems logical to relate the surface change to the entire inversion under clear skies and light winds. Also, the 22 ideal examples show the value of "a" in (1) to approximately equal 1.25. Figure I shows a typical clear night inversion. In order to evaluate the total area (A) as shown in Figure I, we must divide it into two unequal parts as shown in Figure 2. First:

(3)
$$A_1' = \int_0^{(T_0 - T_{mi})} (a^{\Delta T} - 1)\, d(\Delta T)$$

For A_2^I some definitions are made. The slope of any sounding extrapolated to the ground to T_L is assumed here to be such that some height Z will equal twice the base of the triangle ZT_LT_o as shown in Figure 2. Therefore:

(4)
$$A_2' = \frac{Z^2}{4}$$

Now the sum of the areas A_1^I and A_2^I can be expressed:

(5)
$$A' = \int_0^{(T_0 - T_{mi})} (a^{\Delta T} - 1)\, d(\Delta T) + \frac{Z^2}{4}$$

Integration of (5) gives:

$$(5a)\quad A' = \frac{a^{(T_o - T_{M1})} - 1}{\log_e a} - (T_o - T_{M1}) + \frac{Z^2}{4}$$

Equation (5a) is still not in usable form. However, we know,

$$(6)\quad T_o = T_L - \frac{Z}{2}$$

$$(7)\quad Z = a^{(T_o - T_{M1})} - 1$$

By substituting (6) and (7) into (5a) we arrive at,

$$(8)\quad A' = \frac{Z}{\log_e a} + \frac{Z}{2} + \frac{Z^2}{4} - (T_L - T_{M1})$$

By rearranging and substituting into the general quadratic formula,

$$(9)\quad Z = -2\left(\frac{1}{\log_e a} + \frac{1}{2}\right) + 2\left[\left(\frac{1}{\log_e a} + \frac{1}{2}\right)^2 + (T_L - T_{M1}) + A'\right]^{\frac{1}{2}}$$

Since "a" already has been said to equal 1.25 and A' has been found to equal 3.5 $(T_L - T_{M1})$, we can combine and substitute to obtain:

$$(10)\quad Z \approx -10 + 2\left[25 + 4.5(T_L - T_{M1})\right]^{\frac{1}{2}}$$

Inversions recorded later in the season with snow cover indicate "a" in (1) to be close to 1.15, so during snow-cover conditions (10) becomes:

$$(11)\quad Z \approx -15 + 2\left[58 + 4.5(T_L - T_{M1})\right]^{\frac{1}{2}}$$

Since Z is equal to C° units, to obtain feet one must multiply Z times 300 because on adiabatic plotting chart D-11 one centigrade degree equals about 300 feet.

Since all fall soundings appropriate to the problem were used to establish the constants and coefficients in the above equations, only winter soundings are at present available for testing equations (10) and (11). (It is hoped that the aircraft sounding program can again be carried on during the late summer and fall of 1971.) Most clear night inversions occur during summer and fall months. Also, soundings were never taken on Sundays. Nevertheless, some verification examples from some winter soundings are presented in Table I, despite lack of truly ideal conditions.

DATE	CHARACTER OF PAST 12 HRS.	OBSERVED DEPTH FEET	COMPUTED DEPTH FEET
11/17/70	P.C. windy most of night	600	600
11/18/70	M.C. windy at times with sprinkle of snow	700	1500
11/20/70	V.C. snow before midnight	1500	2100
12/02/70	V.C. light winds	1800	1600
12/03/70	M. clear	700	1000
12/05/70	High cloudiness	2700	2700
12/08/70	Cloudy, windy at times	200	300
12/10/70	Cloudy, trace snow	1600	1800
12/11/70	Cloudy	1500	1300
12/12/70	P.C. windy most of time	200	600
12/14/70	High and middle clouds	2300	2100

Table I. Comparison of computed depths of morning inversion layers and observed depths for selected winter soundings.

III. TEMPERATURE INFLUENCES UNDER INVERSIONS

Another approach can be developed which verifies (10) and (11) indirectly. If "a" truly equals 1.25 or 1.15 (snow cover), and if we select clear days when it is evident that ideal inversion conditions exist, we should be able to accurately predict a rise in temperature under the inversion because energy units in degree centigrade squares are known [1].

To do this we add a certain number of heating units under the inversion and compare computed and observed temperatures. To accomplish this, we must develop formulae.

Referring to Figure 3, the area A^{II} under the inversion curve, which represents the number of energy units necessary to raise the surface temperature from T_{MI} to some temperature T_{MA}, is enclosed by the inversion curve and a dry adiabat through T_{MA}. This area, A^{II}, can be divided into two portions. The first portion from T_{MI} to T is given by the integral:

$$\int_0^{(T-T_{MI})} (a^{\Delta T}-1)\, d(\Delta T)$$

The remaining portion of A^{II} is enclosed by the triangle $Z T T_{MA}$. Since the slope of an adiabat is approximately 45°, the height Z of the triangle is approximately equal to the base and the area of the triangle can be expressed as:

$$\frac{Z^2}{2}$$

From (1)

$$Z = a^{(T-T_{MI})} - 1$$

and

$$Z^2 = \left[a^{(T-T_{MI})} - 1\right]^2$$

Adding the two portions of A^{II} together:

$$A'' = \int_0^{(T-T_{MI})} (a^{\Delta T}-1)\, d(\Delta T) + \frac{\left(a^{(T-T_{MI})}-1\right)^2}{2}$$

or

(12) $$A'' = \frac{a^{(T-T_{MI})}-1}{\log_e a} - (T-T_{MI}) + \frac{\left(a^{T-T_{MI}}-1\right)^2}{2}$$

By substituting values of T in (12) we, of course, get A^{II} units.

However, to find the temperature T_{MA} as shown in Figure 3 we must use the equation:

$$T_{MA} = T + a^{T-T_{MI}} - 1 \quad (13)$$

Table 2 shows values of $T_{MA}-T_{MI}$ corresponding to selected values of A^{II} using "a" equaling both 1.15 and 1.25.

A^{II}	a = 1.15 $T_{MA}-T_{MI}$	a = 1.25 $T_{MA}-T_{MI}$
2	4.5	4.0
3	6.0	5.0
4	7.0	6.0
6	8.5	7.0
10	10.0	9.0
12	11.5	9.5
16	13.0	10.5
21	14.5	12.0
27	16.0	13.0
35	18.0	14.5
46	20.0	16.0
59	22.0	17.5

Table 2. Values of $T_{MA}-T_{MI}$ corresponding to selected values of A^{II} for a = 1.15 (snow cover) and a = 1.25.

Curves showing the complete range of A^{II} relative to $T_{MA}-T_{MI}$ are plotted in Figure 4. By computing available energy units at noon from [1], we then proceed to the verification.

IV. VERIFICATION OF TEMPERATURE UNDER INVERSIONS

The verification procedure used in this instance is to attempt to predict the noon, or in three cases the maximum temperature at Helena when the station was under an inversion. Since energy units from sunrise until noon or maximum can be computed, it should be easy to predict a noon or maximum temperature provided "a" of the exponential curve is 1.15 or 1.25. Below, in Table 3, are a few examples under clear or cirrus days during 1969.

DATE	ENERGY U*	UNITS** USED		PREDICTED TEMPERATURE	OBSERVED TEMPERATURE	DIFF.
12/25/69	25	15		28	24	4
12/11/69	25	7		24	23	1
12/6/69	25	15		23	20	3
12/3/69	30	18		26	24	2
12/3/69	30	30	Max	30	33	-3
12/2/69	30	18		28	28	0
12/1/69	30	18		29	31	-2
12/1/69	30	30	Max	34	40	-6
11/30/69	30	18		31	29	+2
11/29/69	30	14		34	32	+2
11/29/69	30	25	Max	38	40	-2
11/28/69	30	18		32	30	2

Table 3. Comparison of predicted temperatures under inversion conditions and observed temperatures.

*Energy units available to reach maximum temperature on clear days.

**Estimated energy units needed to reach noon temperature (a maximum temperature in three cases) taking into consideration cloudiness and snow cover.

As Table 3 shows, predicted noon temperatures are, in general, within two or three degrees of observed values.

A more thorough verification has not been possible due to lack of suitable, clear night soundings. However, computations which have been made under close to ideal conditions of essentially clear skies and light winds result in an accuracy which leads to high confidence in the equations developed. Inversions forming under ideal conditions of clear skies and light winds may occur one out of every two or three nights in the Helena valley July through September, but only one out of every six nights the remainder of the year.

V. CONCLUSIONS

It has been shown that the temperature sounding in the very lowest levels on clear, relatively calm mornings in the Helena valley can be approximated rather accurately by an exponential curve. Formulae have been developed from which the depth of the early morning inversion layer may be computed as a function of the observed minimum temperature at Helena airport and a "temperature" which would have prevailed if no inversion had formed. This latter temperature is obtained by extending the upper portion of a nearby temperature sounding (Great Falls, Montana) downward to the surface. Verification of computed heights of inversions shows good accuracy, especially under ideal radiative conditions. For a more thorough verification of the formulae developed in this study, additional summer and fall low-level aircraft soundings in the Helena valley are needed.

Computation of inversion depths for a number of clear nights during the past several summers and falls using the Great Falls sounding and the Helena minimum temperature indicates theoretically that inversion depths in the Helena valley of 1500 to 2000 feet are not uncommon and 3000 feet not too rare. Generally, inversions tend to be deeper in fall than any other time, which is not too surprising. In general, inversions in the Helena valley occur on the majority of all nights, and become strong on an estimated one hundred nights per year.

VI. ACKNOWLEDGMENTS

Appreciation is expressed to Mr. Woodrow W. Dickey, Scientific Services Division, Western Region Headquarters, for his helpful suggestions. Mr. Richard Dightman, Meteorologist in Charge, Weather Service Office, Helena, Montana, was primarily responsible for the arrangements with the National Health Service to continue the low-level aircraft sounding program.

VII. REFERENCES

1. Olsen, David E., "Forecasting Maximum Temperatures at Helena, Montana", Technical Memorandum WSTM WR-43, October 1969.

2. Jefferson, C. J., "Temperature Rise on Clear Mornings", The Meteorological Magazine, Vol. 79, No. 932, February 1950, page 33.

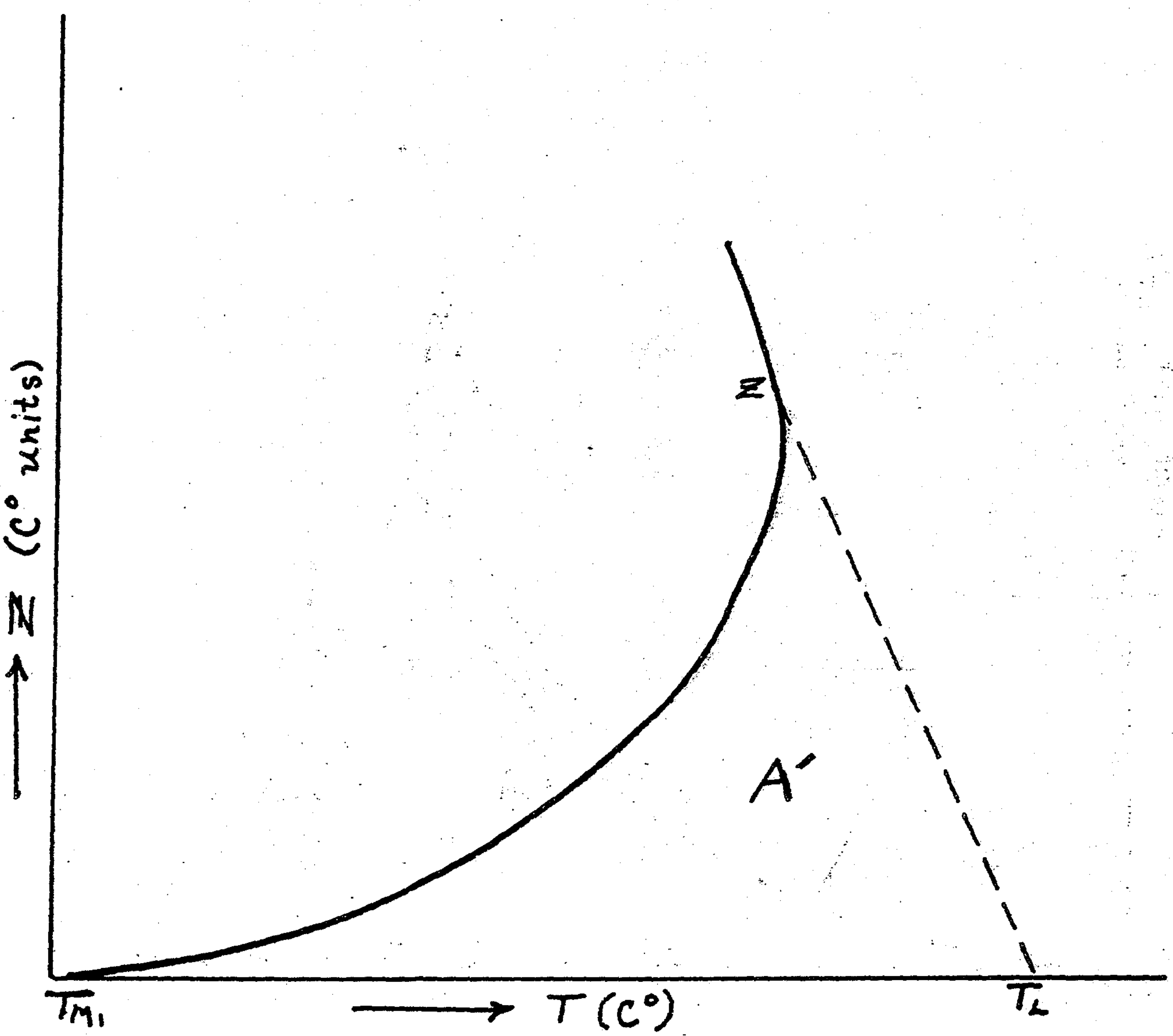

Figure 1. Schematic inversion and the area enclosed by the inversion curve and the approximate normal lapse rate curve extended from height Z to the ground to temperature T_L.

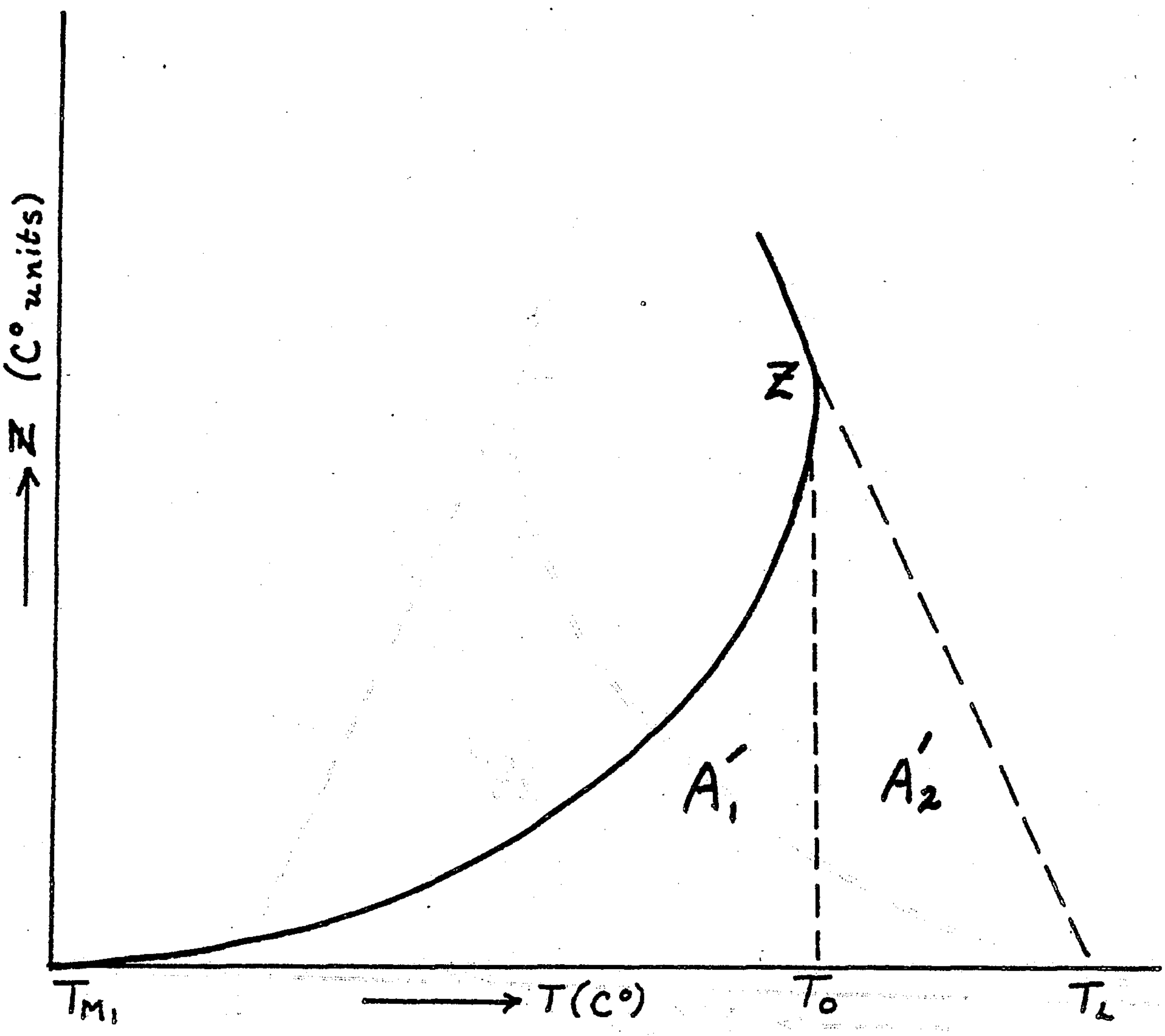

Figure 2. Illustrating the inversion curve divided into two unequal areas in order to derive an equation to find the total area enclosed by T_{Ml} Z T_L.

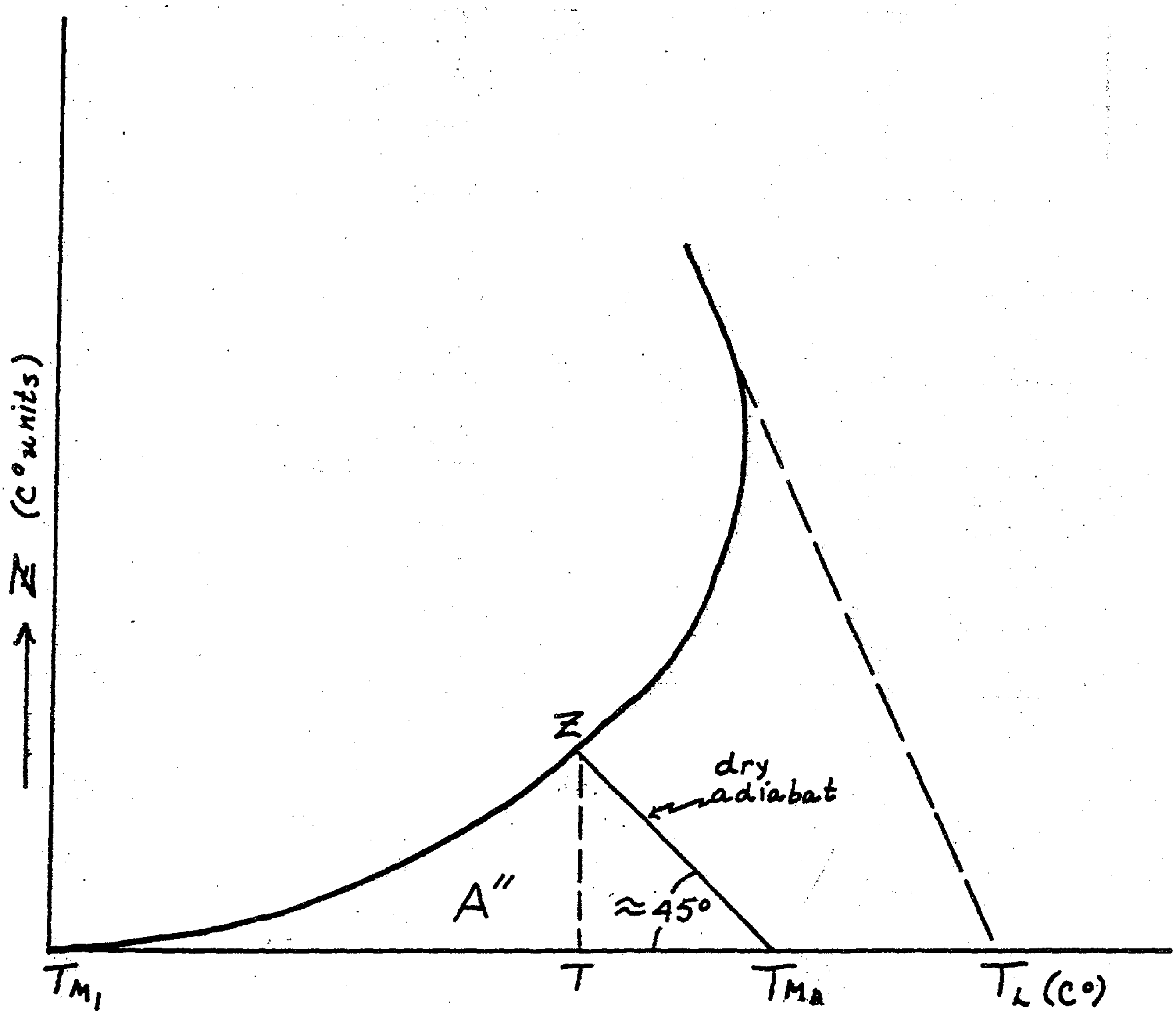

Figure 3. Illustrating the area under the inversion curve representing the energy necessary to raise the temperature from T_{MI} to T_{MA}, and the division of this area into two unequal portions in order to derive an equation to evaluate the area.

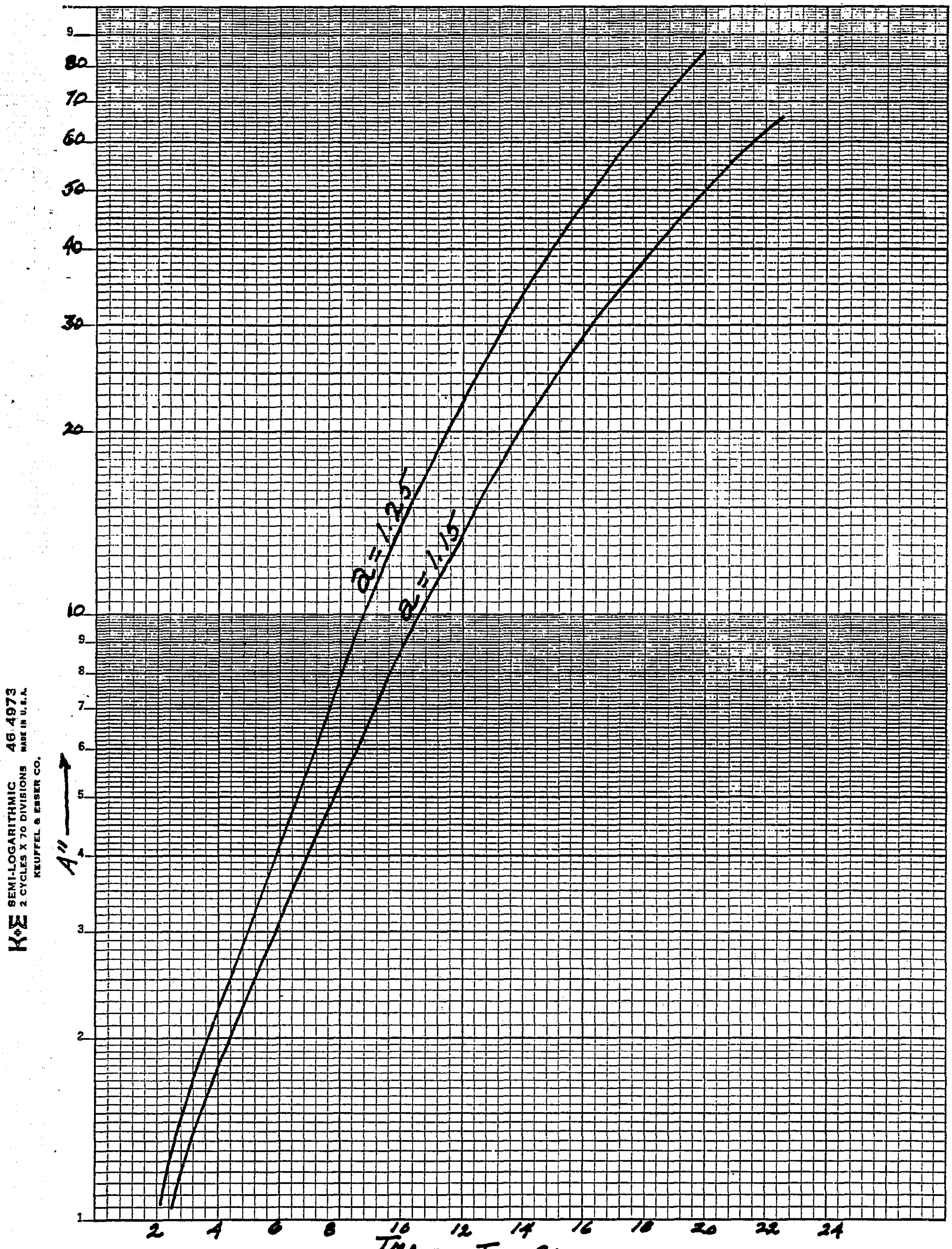

Figure 4. Relationship between A'' and TMA-TMI for a = 1.15 and a = 1.25.

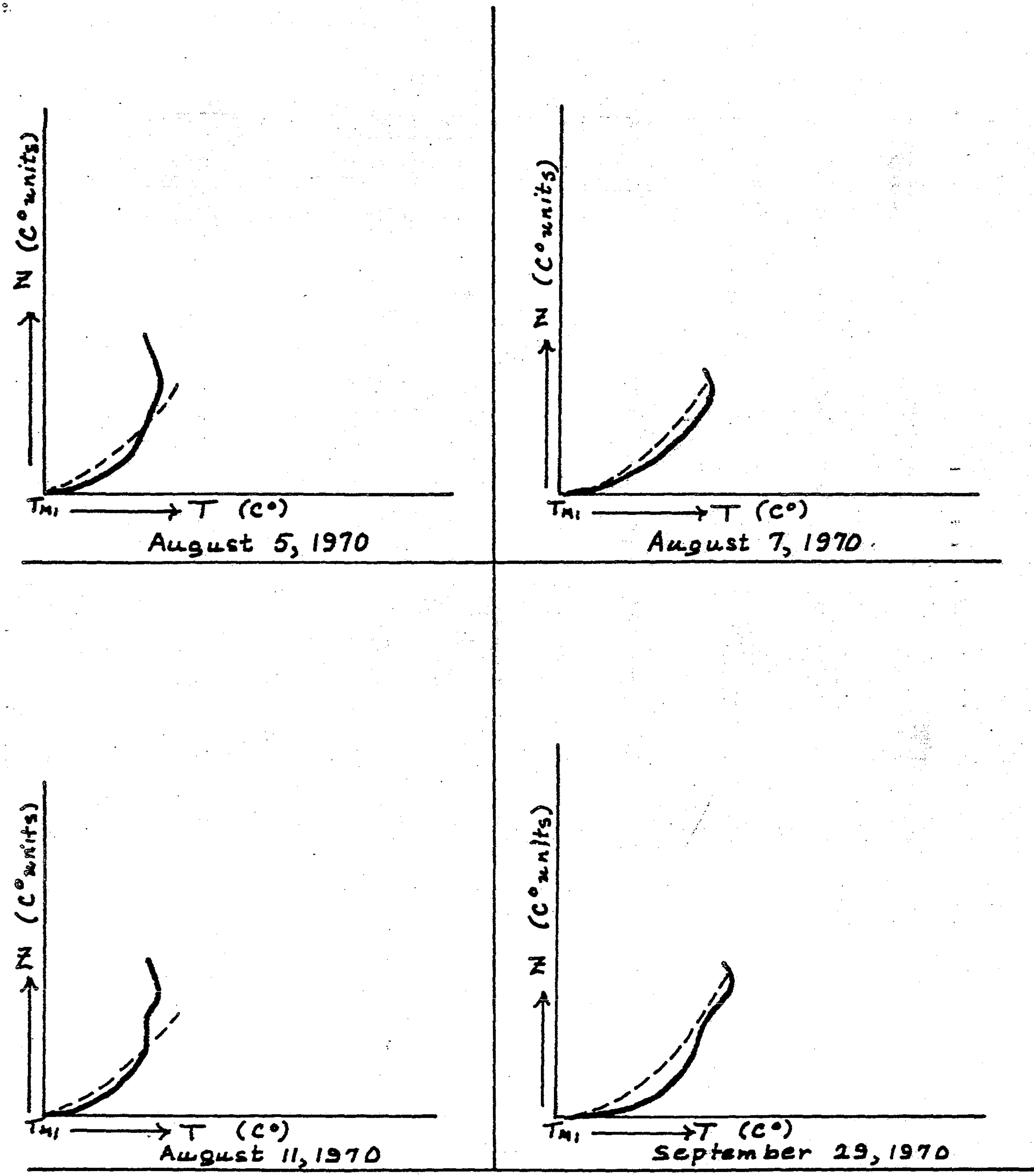

Figure 5. Four examples comparing the actual temperature curve (solid line) with the predicted temperature curve (dashed line).

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