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APPLYING TRENBERTH'S APPROXIMATION OF THE OMEGA EQUATION

The quasi-geostrophic omega equation is a valuable tool for diagnosing areas of synoptic scale vertical motion from common weather maps. The most traditional way of estimating vertical motion is to identify areas of positive (or cyclonic) vorticity advection (PVA) at 500mb. However, the quasi-geostrophic vertical motion in the omega equation is actually related to the PVA increasing with height **and** the Laplacian of temperature advection. Many studies have shown that the temperature advection term can be as important to the overall vertical motion field as the PVA term and, in many cases, may cancel the contribution from the PVA term. Furthermore, the 500mb PVA pattern doesn't always look the same as the pattern of where PVA is increasing with height (for example, Durran and Snellman, 1987). Thus, several ways of interpreting the complete quasi-geostrophic omega equation that do not rely on a two-term approach have been developed.

Although the Q-vector method is accurate (see WRTA 90-07), it is difficult to apply with current forecast maps available on AFOS. Perhaps the easiest to use with forecast maps that are currently available on AFOS is the Trenberth approximation (Trenberth, 1978). Trenberth shows that the advection of positive (cyclonic) vorticity by the thermal wind is a relatively good approximation to the sum of the PVA and temperature advection terms of the full omega equation. This approximation is most valid in the middle atmosphere between about 600mb and 400mb.

Ideally, charts of the thermal wind and vorticity at 500mb would allow the easy identification of areas of vertical motion consistent with Trenberth's approximation to the omega equation. Unfortunately, the necessary charts are not currently available on AFOS. Forecasters often use the 1000-500mb thickness chart and the 500mb vorticity chart to approximate Trenberth's method. The 1000-500mb thickness chart shows the thermal wind between 1000mb and 500mb. Yet, in the truest application of Trenberth's approximation, we need to know the thermal wind in a small layer near 500mb (say, between 600 and 400mb). The 1000-500mb thermal wind is likely to be different than the 600-400mb thermal wind in areas where there are large changes in the wind direction with height. Unfortunately, areas where the wind direction changes with height are those areas where a baroclinic system is developing and where estimates of vertical motions are crucial. Thus, using the 1000-500mb thickness with the 500mb vorticity may not be as appropriate during those situations when it is most critical to have accurate vertical motion information. The thickness and vorticity approach is usually better than the typical heights and vorticity approach, but it is not a perfect indicator of the quasi-geostrophic vertical motion field.

However, there are AFOS charts available that can provide more information than just the 1000-500mb thickness and 500mb vorticity. Figure 1 shows the 12-hour NGM forecasts of 1000-500mb thickness and 500mb vorticity valid at 0000Z on October 30, 1991. The 1000-500mb thermal wind is parallel to the thickness contours and, thus, positive vorticity advection by the thermal wind is indicated in southwestern Idaho, northeastern Nevada as well as all of Utah and northern Arizona. However, Fig. 2 shows the 500mb trough was forecast to be over northeastern Nevada while Fig. 3 shows the surface trough was forecast to be over southern Utah. This tilt of the trough line with height implies changes in the wind direction with height as well as changes in the thermal wind with height and, therefore, the estimate of Trenberth's approximation via Fig. 1 is in doubt (it is already somewhat in doubt since using 1000-500mb thickness involves the lower atmosphere where Trenberth finds that his approximation is less valid).

We can gain some useful information by considering the 700mb height field available on AFOS. Where there is a maximum in curvature of the height lines, there is a maximum in geostrophic vorticity, and we can at least approximate the full 700mb vorticity by the 700mb geostrophic vorticity. Thus, by overlaying the 1000-500mb thickness with the 700mb height, we have an estimate of the thermal wind and vorticity at 700mb. We can apply Trenberth's approximation at 700mb and hope that it is similar to what we would find at 500mb (where Trenberth has shown that the approximation is more valid). Figure 4 shows the 12-hour NGM forecasts of 1000-500mb thickness with the 700mb height overlaid. The 700mb trough line (maximum in curvature) runs from the low center over western Utah, southwest to southern California. By assuming that a maximum in 700mb vorticity also lies along this line, we can conclude that there is positive vorticity advection by the thermal wind over Utah, Colorado, and northwestern Arizona, but negative vorticity advection over northeastern Nevada and southern Idaho. Figure 5 shows a satellite picture valid near 0000Z and shows that clouds with the coldest tops were concentrated in Utah and Colorado (similar to the advection pattern shown in Fig. 4) rather than in Nevada and Idaho (as indicated by the pattern shown in Fig. 1).

In cases of developing cyclones where changes of wind direction with height are large, overlaying 700mb height with 1000-500mb thickness may help visualize Trenberth's approximation to the omega equation, rather than overlaying 500mb vorticity with 1000-500mb thickness. Fields of analyzed 700-300mb thickness can be generated by Mike Foster's Southern Region PC programs UA and AUA, and overlaid with 500mb vorticity. This approach gives the true Trenberth approximation to the omega equation. However, for forecast charts, 700-300mb thickness is not readily available at the current time, and estimating the 700mb vorticity may prove to be valuable. The difficulty in estimating the 700mb vorticity from the curvature of the 700mb height lines is certainly a hindrance to this technique, and may make the daily application of this technique difficult. However, in developing situations where there is considerable tilt of a trough with height, it may prove to be helpful.

References

Durran, D. R. and L. W. Snellman, 1987. The diagnosis of synoptic-scale vertical motion in an operational environment. *Weather and Forecasting* 2: 17-31.

Trenberth, K. E. 1978. On the interpretation of the diagnostic quasi-geostrophic omega equation. *Monthly Weather Review* 106: 131-137.

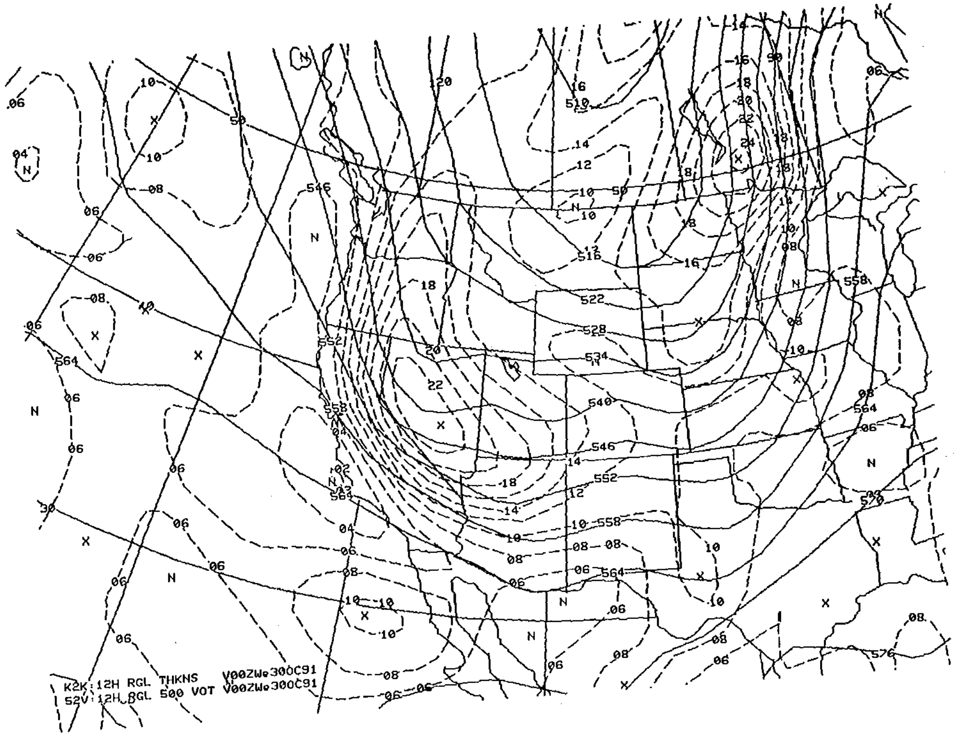


Fig. 1

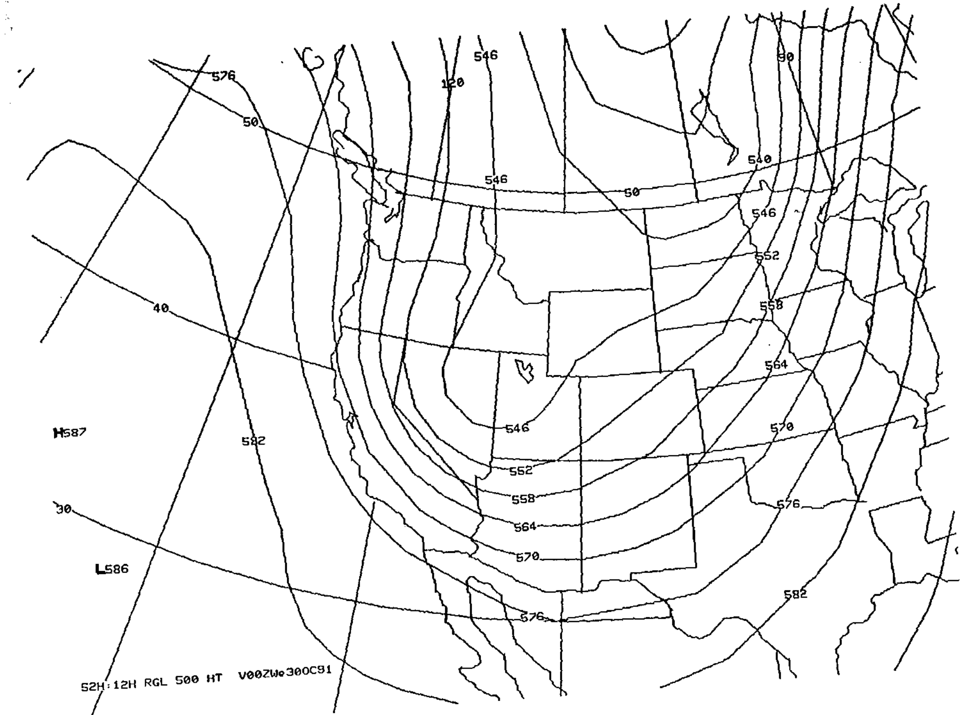
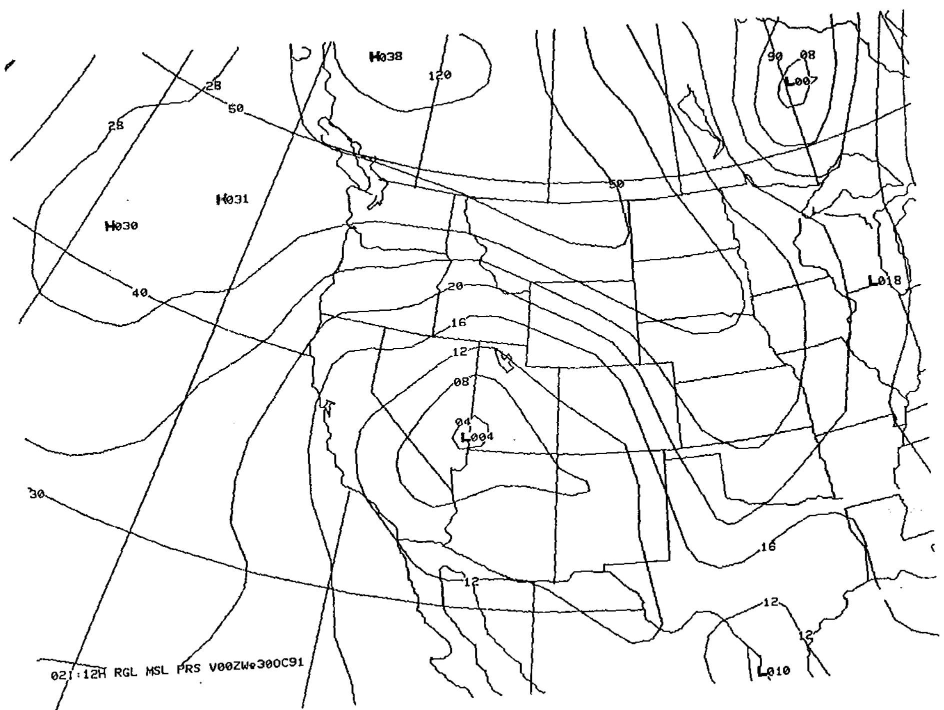


Fig. 2



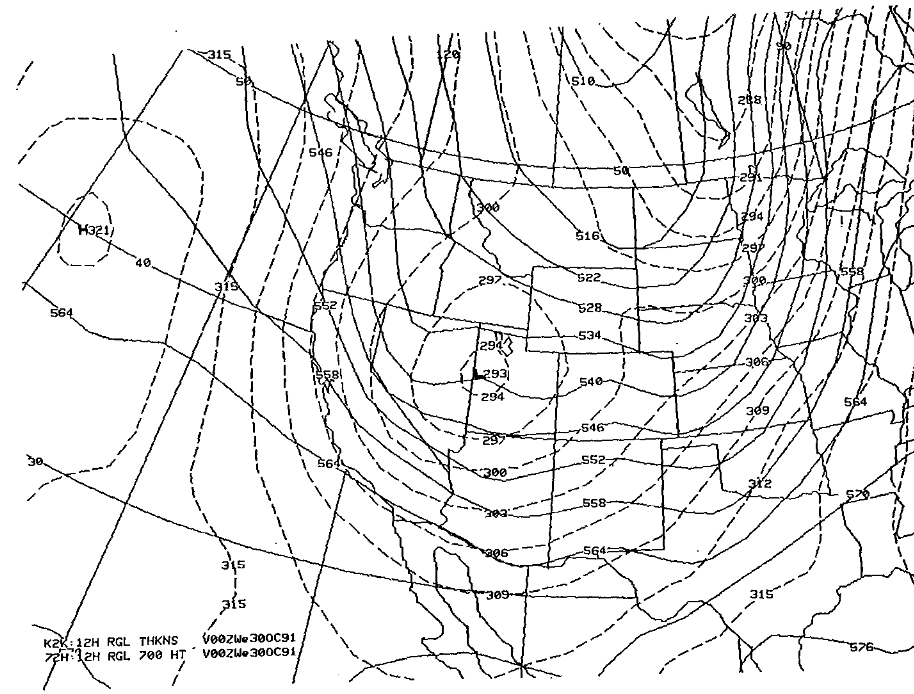


Fig. 4

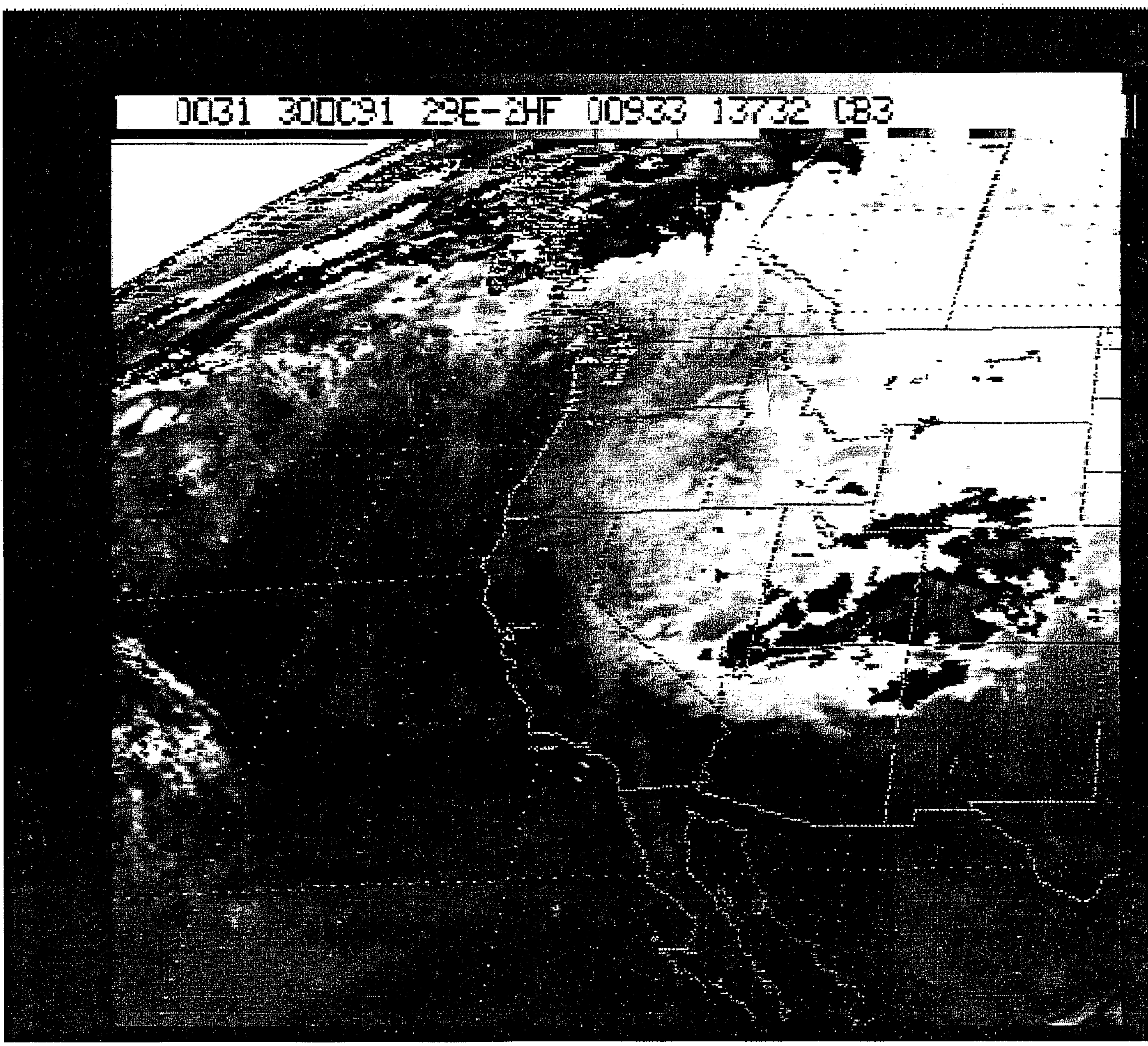


Fig. 5